

THE DENSIFICATION AND DIAGENESIS OF SNOW

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Introduction

The Greenland and Antarctic Ice Sheets are monomineralic rock formations, primarily metamorphic, but with sedimentary veneers. The metamorphic part consists of glacier ice that has been metamorphosed primarily by flow caused by unbalanced stresses. The sedimentary veneer has a maximum thickness of about 90 meters and consists of snow and firn.* This paper is concerned with diagenetic processes occurring within the sedimentary veneer, causing evolution of loose snow to glacier ice. This diagenesis constitutes one step in the over-all balance between accumulation at the snow surface and loss of ice by flow within the metamorphic part of the ice sheet. Although the discussion is based partly on field observations from Greenland, it deals with the general problem of snow densification; indeed, the results are not completely limited to rocks composed of ice but apply in part to the over-all diagenesis of unconsolidated sediments into consolidated sedimentary rock.

Because of the large mass of snow density data accumulating from Greenland, Antarctica, and temperate glaciers, it is important to have parameters that are descriptive of the in situ conditions of the snow. To permit comparison of data from

* Snow is sometimes referred to as firn after it has survived a melt season. However, there is no physically defined distinction between the two materials. The question of defining a physical difference between snow, firn, and névé is discussed herein. The physical difference between snow (or firn or névé) and glacier ice is best expressed in terms of permeability: snow (or firn or névé) is permeable to air flow because it consists of ice grains with intercommunicating pore space, whereas glacier ice consists of ice with isolated air bubbles and is not permeable to air.

various regions, it is also important to have a simple analytical expression for depth-density profiles which incorporate these parameters. Our aim is to determine a depth-density relation that is as general as possible with as few parameters as possible.

The densification of snow is analogous to the sintering process in powder metallurgy. In each case a powder is transformed into a solid mass by eliminating pore space and reducing surface area. The diagenetic processes within the sedimentary veneer of the Greenland Ice Sheet constitute a gigantic "sintering experiment." However, this experiment takes place under conditions somewhat different from those of the laboratory. The primary difference is in the manner of controlling the variables: pressure, temperature, volume, and time. Most laboratory experiments are of very short time duration when compared with this natural experiment.

Time enters the densification problem in several ways on the ice sheet. As a first approximation to an analytic expression for this problem, some simplifying assumptions are made. First it is assumed that accumulation of new snow occurs at a constant rate " A " g/cm² per year which is slow compared with the time required for snow to react to the applied stress.* Thus, within the sedimentary veneer a steady-state system exists with the snow and firn moving downward through a vertical stress gradient as they densify, while the depth-density curve remains invariant with time, as stated by Sorge's law (2) and as shown by Benson (1). Thus, a portion of the time dependence of densification is eliminated as an explicit variable because of the steady-state conditions. Time also enters in the form of truly time-dependent processes that will also be discussed.

It is also assumed that the ice composing the grains remains at constant density;† and as a direct result of this the observed volume changes are caused solely by the elimination of pore space. Thus, the specific volume of the ice involved in our

* The large-scale flow of glacier ice caused by unbalanced stresses do not enter this discussion. Herein, we concentrate on the diagenesis of snow and firn within the sedimentary veneer. If an ice sheet is in equilibrium, the amount of material lost each year by flow at depth, in the metamorphic part, will equal the amount added at the surface; in this case the surface altitude profile of the ice sheet is invariant with time. The Greenland ice sheet is essentially in equilibrium (1).

† This assumption is reasonable because the maximum pressure at the base of the ice sheet does not exceed 300 atmospheres, and this produces an elastic increase in ice density of the order of 0.005 g/cm³.

discussion is constant $\approx 1.09 \text{ cm}^3/\text{gm}^{-1}$. Also, at a given point on the ice sheet the temperature below 10-m depth is essentially invariant with time.

The remaining explicit variables are pore space and vertical stress, i. e., load of overlying snow and firn. Under the steady-state, constant-temperature conditions of the experiment it will be assumed that the rate of elimination of pore space as load increases is directly proportional to the amount of pore space present; i. e.,

$$\frac{dv_p}{d\sigma} = -mv_p$$

or

$$\frac{dv}{d\sigma} = -m(v - v_i) \quad (1)$$

where

$$v = \frac{1}{\rho} = \text{specific volume of snow or firn } (\rho = \text{density})$$

$$v_i = \text{specific volume of ice } (1.09 \text{ cm}^3/\text{g})$$

$$v_p = (v - v_i) = \text{volume of pore space}$$

$$\sigma = \int_0^z \rho \, dz = \text{load at depth } z \text{ below snow surface}$$

and m is a function of the mechanism of compaction, including time-dependent, and stress-dependent terms. The functional nature of these mechanisms is unknown, but under the assumed conditions of a steady-state system with constant rate of accumulation, m will be treated as a constant.* This permits immediate examination of the first approximation just outlined and expressed in Eq. 1. In particular, the consequences of this approach will be compared with observation.

Equation 1 may be readily solved for v in terms of σ to obtain

$$v = v_i + (v_0 - v_i) e^{-m\sigma} \quad (2)$$

*The parameter m is probably temperature dependent but, since the temperature gradient and variation are slight at a given location on the ice sheet, it is assumed that temperature is constant. The annual temperature amplitude is only 5°C at depth of 3 meters, 1°C at 7 meters, and approximately 0.3°C at 10 meters below snow surface (1).

where v_0 is the specific volume when $\sigma = 0$. This sort of law is quite common in many physical processes of this general type.

Depth-density data from SIPRE's research station* 2-100, near the dry-snow line† in northwest Greenland, and from the soaked facies† of two temperate glaciers are expressed in terms of volume and load in Fig. 1. The curves were computed from Eq. 2 with the values for the constants given in Table 1. Values of m for other stations in the nonsoaked facies of Greenland are

Table 1. Constants for Depth-Density Data

	Load (g/cm ²)	m (cm ² /g)	v_0 (cm ³ /g)
Station 2-100 Greenland	$0 < \sigma < 455$	16.0×10^{-4}	2.65
	$455 < \sigma < \infty$	4.3×10^{-4}	2.00
Upper Seward and Blue Glaciers	$0 < \sigma < 320$	37.5×10^{-4}	2.12
	$320 < \sigma < \infty$	12.0×10^{-4}	1.56

in good agreement with those of station 2-100. From Fig. 1 it is apparent that the rate of densification in soaked facies is more than twice as great as in nonsoaked facies (m values from soaked facies in Greenland agree with those shown for the Upper Seward

* Data at this station are averaged from measurements made by one of the writers in 1953, 1954, and 1955 (1), from measurements made by other SIPRE personnel in the deep pit of 1954, and from the 400-m core.

† The facies classification of glaciers presented by Benson (1) subdivides glaciers into four different regions according to measurements of temperature, density, and ram hardness. The "soaked facies" lies between the firn line and the saturation line, the annual increment of snowfall is completely wetted by melt water in this facies. The two nonsoaked facies lie above the saturation line and are subdivided at the dry-snow line. Between the saturation line and the dry-snow line the upper part of the annual accumulation increment may be wetted, but the entire year's layer is neither wetted nor raised to 0°C, this is the percolation facies. The dry-snow line marks the upper limit (altitude) of surface melting; the dry-snow facies includes all of the glacier lying above the dry-snow line, and negligible melting occurs in it.

and Blue Glaciers (1); this is attributed to the presence of melt water throughout the snow cover of the soaked facies. Herein, the discussion is concentrated on cases where melting is negligible, in particular, the data from station 2-100 will be discussed in detail because they are the most complete.

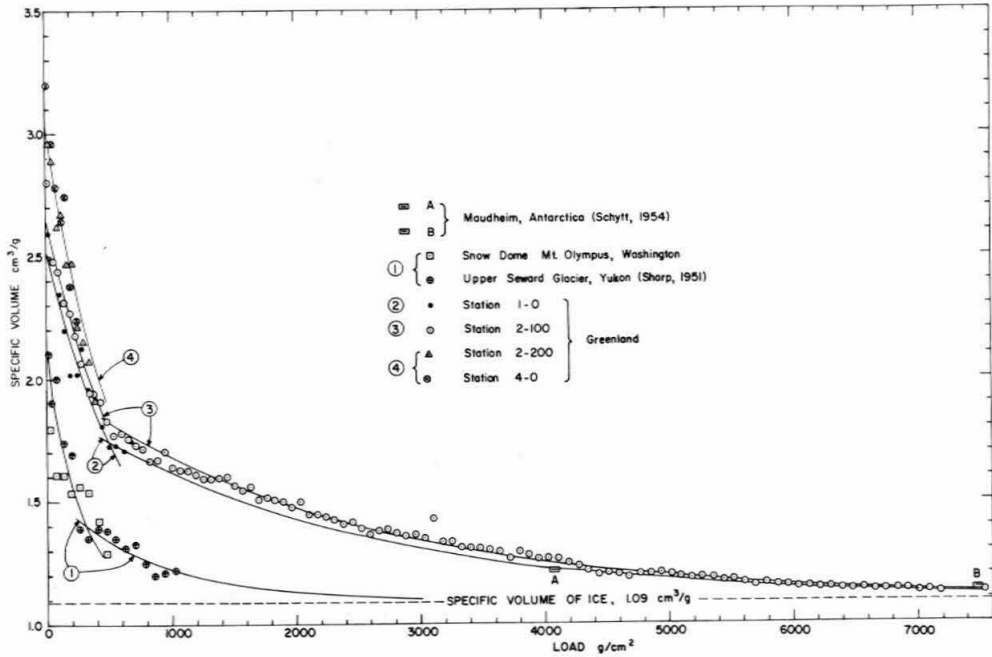


Fig. 1. Specific volume versus load. Soaking is negligible at station 2-100 in Greenland, whereas complete soaking of the entire annual accumulation occurs on the Snow Dome of Mt. Olympus and on the upper Seward Glacier.

At station 2-100, a change in rate of densification about 10 meters below snow surface is apparent in Fig. 1. For convenience, the depth at which this discontinuity occurs is called the "critical depth" z_c , and the load, density, specific volume, porosity, void ratio, and temperature values measured at this depth will be referred to as the critical values σ_c , ρ_c , v_c , ϵ_c , and T_c , respectively. The slope of the load volume curve is given by Eq. 1 both above and below z_c , but as seen in Table 1, the parameter m above z_c is nearly four times greater than it is below. The abrupt change in rate of densification is observed in other data from Greenland (1), and as seen in Fig. 1, it occurs at a shallower depth at higher density values in soaked than in nonsoaked facies.

Before discussing mechanisms of densification, we shall present the depth-density curve that results from our steady-state assumption. The reason for examining the depth-density curve is that it related directly to the measured variables of depth and density. The explicit depth-density relationship is obtained from Eq. 1 by making the following substitutions:

$$v = \frac{1}{\rho}, \quad dv = -\frac{d\rho}{\rho^2}, \quad \sigma = \int_0^z \rho \, dz, \quad \text{and} \quad d\sigma = \rho \, dz$$

The resulting depth-density equation (1) is

$$z = \frac{1}{m\rho_i} [K - (\epsilon + \ln \epsilon)] \quad (3)$$

where

$$K = \frac{\rho_i - \rho_0}{\rho_0} + \ln \frac{\rho_i - \rho_0}{\rho_0} = \epsilon_0 + \ln \epsilon_0$$

$$\epsilon = \frac{\rho_i - \rho}{\rho} = \text{void ratio for firn of density } \rho$$

and

$$\epsilon_0 = \frac{\rho_i - \rho_0}{\rho_0} = \text{void ratio for firn of density } \rho_0$$

In Fig. 2, Eq. 3 is compared with data from station 2-100

It is of interest to examine the nature of the depth-density curve as revealed by the derivatives $d\rho/dz$ and $d^2\rho/dz^2$. The first derivative states that the change in density with depth is

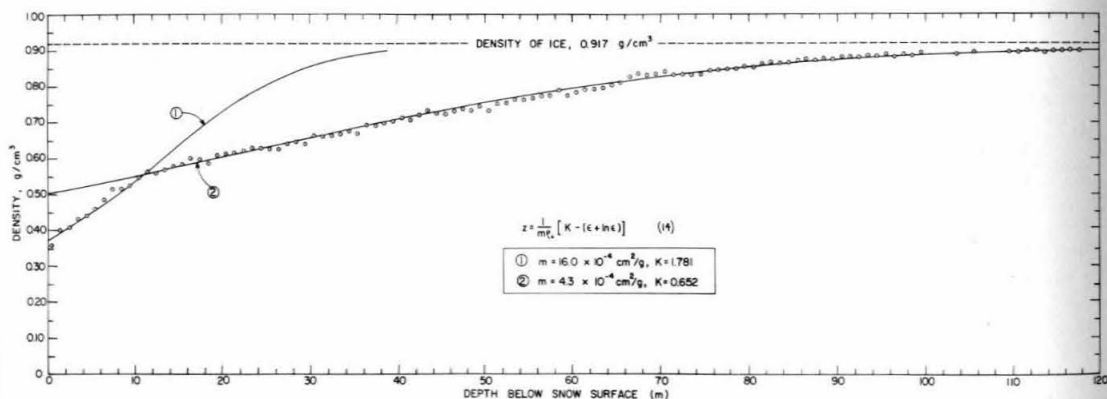


Fig. 2. Depth-density data from station 2-100 in Greenland compared with Eq. 2.

proportional to the product of porosity and the square of the density; i. e.,

$$\frac{d\rho}{dz} = m\rho^3 \left(\frac{1}{\rho} - \frac{1}{\rho_i} \right) = m\rho^2 \left(\frac{\rho_i - \rho}{\rho_i} \right) = m\rho^2 n \quad (4)$$

where

$$n = \text{porosity} = \frac{\rho_i - \rho}{\rho_i}$$

The second derivative is

$$\begin{aligned} \frac{d^2\rho}{dz^2} &= m \frac{d}{dz} \left(\rho^2 - \frac{\rho^3}{\rho_i} \right) \\ &= \left(\frac{m}{\rho_i} \right)^2 \rho^3 \left(2\rho_i - 3\rho \right) (\rho_i - \rho) \end{aligned} \quad (5)$$

Several interesting points about the curvature of the depth-density curve revealed by Eq. 5 are now summarized:

- a. $\frac{d^2\rho}{dz^2} = 0$ when $\rho = \frac{2}{3}\rho_i \approx 0.61 \text{ g/cm}^3$
- b. $\frac{d^2\rho}{dz^2} > 0$ when $\rho < \frac{2}{3}\rho_i$

and

- c. $\frac{d^2\rho}{dz^2} < 0$ when $\rho > \frac{2}{3}\rho_i$

The point of inflection at $\rho = \frac{2}{3}\rho_i$ is independent of the value of the parameter m but is a direct consequence of the functional relationship assumed in Eq. 1.

The over-all curvature of the depth-density curve is slight; therefore, the existence of a point of inflection demands that the curve be nearly linear for some distance on either side of it. The observed depth-density data are nearly linear over the depth range of 10 to 50 meters but nonlinear above and below. The nearly linear part of the curve has made it difficult to derive a simple functional relationship between depth and density that holds for the entire depth range. As stated by Landauer (1959),* "...So

* Also see the discussions in SIPRE Technical Report 20 and Research Reports 26 and 70.

far we have not been able to express this relationship in any simple yet accurate way." Actually, there is no a priori reason why depth should be a significant variable. Compressive stresses on the firn increase with depth, but because the firn density is not constant, we cannot replace the load at a given depth by the depth itself. The approach used herein allows the problem to be stated as changes in pore space produced by increasing load; the solution of this problem, checked against measurement, is then transformed into an explicit depth-density relationship. The point of inflection with the second derivative being positive for $\rho < \frac{2}{3}\rho_i$ and negative for $\rho > \frac{2}{3}\rho_i$ shows clearly in Fig. 2.

A physical reason for the existence of a point of inflection is that the model does not allow negative densities. Thus, from a purely mathematical point of view, the depth-density curve obtained from the steady-state assumption expressed in Eq. 1 is asymptotic to zero density as z approaches $-\infty$, and asymptotic to the density of pure ice as z approaches $+\infty$; with $z = 0$ at the snow surface. This depth-density curve explicitly includes a nearly linear relation between depth and density in the depth range $10 < z < 50$ meters, and this agrees well with observation.

As pointed out above, the parameter m actually represents a lumping of time- and stress-dependent terms. As a result of assuming steady-state conditions at a given point, m was treated as a constant to simplify the mathematical treatment. The agreement between observation and the curve obtained from this simplifying assumption encourages one to accept the steady-state model for further discussion. Deviations from steady-state conditions at a given point will occur only with changes in temperature and/or rate of accumulation, and it is reasonable to expect such changes to be gradual. However, the effect of a controlled variation in temperature and/or accumulation may be easily investigated by making studies at points in the ice sheet, or on glaciers in general, which have any desired values of these variables.

Mechanisms of Densification

By way of introduction, the problem of snow densification on the Greenland Ice Sheet has been presented as a huge sintering experiment. This experiment is always in progress, and we think of it as being run at constant temperature with the snow moving through a vertical stress gradient at a constant rate, i. e., under steady-state conditions. Under these "experimental conditions" the assumption of a simple relationship between pore space and load agrees well with observation. Thus, it is possible to express the observed depth-density

relationships by a simple equation. Up to this point the possible mechanisms of densification have not been discussed. The purpose of this section is to discuss them and attempt to determine the dominant ones.

Before discussing mechanisms it is useful to call attention to a major feature of Figs. 1 and 2, namely, the change in rate of densification at the critical density. This change occurs regardless of whether or not significant wetting occurs. We may reasonably expect it to be associated with a change in the dominant mechanism of densification. Since the change constitutes a marked reduction in rate of densification, we should expect to find that one or more mechanisms are either markedly reduced in effectiveness or eliminated, as the critical density is exceeded.

There are at least seven mechanisms of material transport which may lead to densification:

1. Melting and refreezing.
2. Grain packing (into the closest possible arrangement).
3. Evaporation condensation (this may occur with either liquid or solid surfaces exposed, in the latter case the combined process is termed sublimation).
4. Surface or grain-boundary diffusion of molecules.
5. Volume diffusion of molecules, or
6. Plastic flow, and
7. Viscous flow.

These processes are not mutually exclusive, and any or all of them may act simultaneously in new snow. However, some mechanisms are more effective than others, and some may be completely eliminated during various stages in the diagenetic history. The most obvious example of this is the elimination of item 1, i. e., melting and refreezing, at temperatures below 0°C. Where melting is negligible during the entire diagenetic history, the over-all densification rates are less than half of the rates observed where melting does occur; this shows clearly in Fig. 1. Another example is the case of grain packing. Snow density increases by closer packing of the grains, but there is a limit beyond which grains cannot be more tightly packed without breaking or deforming.

Densification of Snow with Less than Critical Density

Grain packing, enhanced by the grain-rounding action of other mechanisms, apparently dominates the first stages of densification. For simplicity we shall first consider the case where melting does not occur, and inquire as to the limit of densification possible by packing alone. In the special case of homogeneous spheres the theoretical extremes are

1. Cubical arrangement or loose packing, 47.64% porosity.

2. Rhombohedral arrangement or close packing, 25.95% porosity.

However, these theoretical extremes are not realized experimentally. Indeed it has been repeatedly demonstrated

"...that assemblages of spheres, or even sand particles, will have porosities averaging about 40 per cent in spite of careful efforts to induce closer packing, and even though the predominant array in the assemblage is rhombohedral with a porosity of only 26 per cent" (Muskat, 1937, p. 13).

The average porosity of 40% corresponds to a snow density of 0.55 g/cm^3 . The minimum recorded porosity resulting from extensive jarring, combined with piston tamping of particles is 36 to 37% (3-5). This corresponds to a snow density of about 0.58 g/cm^3 and compares well with maximum packing densities observed in the laboratory (6). These values lie approximately midway between the theoretical extremes for packed spheres and correspond to the observed critical density in dry snow. The critical porosity may be regarded as a close random packing of particles.

It is proposed that the critical porosity, or critical density, as seen in Figs. 1 and 2, represents the limit beyond which grain packing is no longer effective. The loss of this mechanism gives rise to the fourfold reduction in rate of densification observed as the critical density is exceeded. Compaction beyond the critical density proceeds solely by mechanisms that cause grains to change size and shape as they grow together to relieve stress.

The preceding discussion was restricted to dry snow. The effect of free water in snow is to permit closer packing than is possible in dry snow. Besides filling some of the interstices, water rounds, lubricates, and buoys the grains permitting increased freedom of rearrangement; also, surface tension of the water tends to pull the grains together. The net result is an increase in the rate of densification by packing, and a higher critical density, for wet than for dry snow as shown in Fig. 1. Thus, it seems that the above discussion may be applied to wet snow as well as to dry snow.

The critical density represents a more fundamental division of properties than does the transition from snow (or firn) to glacier ice. It is associated with a structural change in the material, whereas the transition from snow (or firn) to glacier ice is gradual (see Figs. 1 and 2) and, by definition, occurs when air permeability becomes zero. Because the critical density represents a structural change it should be reflected in other physical properties. A rapid transition in mechanical properties of dry snow does indeed occur at the critical density; on bilog coordinates many physical properties can be represented over an appreciable range by two lines intersecting at approximately 0.55 g/cm^3 (Fig. 3). Seismic data indicate a discontinuity at a depth of about

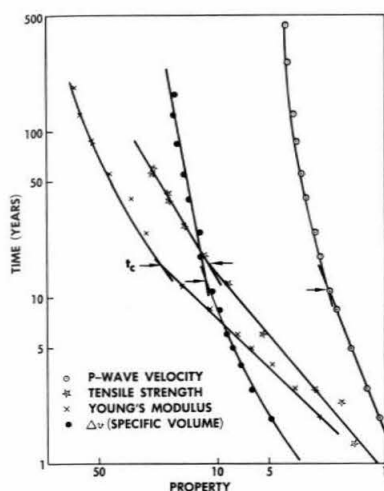


Fig. 3. Various physical properties of snow and firn versus the age of the layer from which they were obtained. The arrows represent the critical time. Data are from Bentley et al. (1957), Butkovich (1958), Nakaya (1958), and Langeway (1958).

14 meters in nonsoaked snow near SIPRE's test station in north-west Greenland (7). The density of 0.57 g/cm^3 at this depth is essentially the critical density. The critical density also compares well with maximum densities obtained by artificial compaction of snow in the field and laboratory.

Densification of Snow with Greater than Critical Density

When the ice grains, comprising snow or firn, are packed as tightly as possible, i.e., when they have reached, or exceeded the critical density, further densification must proceed by changes in size and shape of the grains. For convenience, we continue to restrict the discussion to dry material, recognizing, as above, that the effect of melting and refreezing is to increase the overall rate of densification. The closely packed ice particles relieve stress, acting across their contact boundaries, by growing together. The rate of growth depends on the mechanism of material transport. If a circular contact between grains is assumed, the rate of contact varies with time as follows (8, 9):

a. For viscous yielding,

$$r^2 = At \quad (6)$$

$$A = \frac{3\gamma R_0}{2\eta}$$

b. For evaporation condensation,

$$r^3 = Ct \quad (7)$$

$$C = \frac{3\pi M(M/2\pi RT)^{\frac{1}{2}} \gamma P_0 R_0}{d^2 RT}$$

c. For volume diffusion,

$$r^5 = Dt \quad (8)$$

$$D = \frac{a_1 \pi \gamma \delta^3 R_0^2}{kT} D_v$$

d. For surface diffusion,

$$r^7 = Et \quad (9)$$

$$E = \frac{a_2 \sigma R_0^3 \delta^4 \gamma}{kT} D_s$$

where

r = radius of contact	M = molecular weight
t = time	d = density of ice
T = absolute temperature	R = gas constant
R_0 = original radius of spheres	δ = interatomic distance
γ = surface energy	D_v = coefficient of volume diffusion
σ = surface tension	D_s = coefficient of surface diffusion
η = viscosity	k = Boltzmann constant
P_0 = vapor pressure over flat ice surface	a_i = numerical constants

For this preliminary study we shall accept uncritically the complete applicability of the above equations to ice. It should be pointed out that Kingery and Berg (10) and others have determined that volume and surface diffusion may proceed at the same rate. It is difficult to give an analytic solution for surface diffusion because the migration of molecules depends on the geometry of diffusion paths followed. However, in either case of diffusion, r varies according to the fifth or greater root of t .

We shall proceed to deduce the probable mechanisms of material transport in ice from indirect evidence. Some data are available on the rate of change of certain properties of snow with time, namely, strength, hardness, and dielectric constant. In order to utilize these data to infer growth mechanism we must relate them to contact area between grains. It is reasonable to assume that any stress applied to a mass of snow is distributed over only the contact areas. This makes most mechanical properties of snow directly proportional to the contact area between grains. The radius of contact area between grains is related to time by the preceding equations; therefore, the variation with time of physical properties provides an indirect measure of the variation of contact area with time. Since the rate of increase of contact area also depends on the growth mechanism involved,

one can speculate as to which mechanism predominates by examining the time dependency of physical properties of snow. We begin by recognizing that evaporation-condensation and surface diffusion cannot lead to over-all densification because grain centers are not allowed to approach one another.

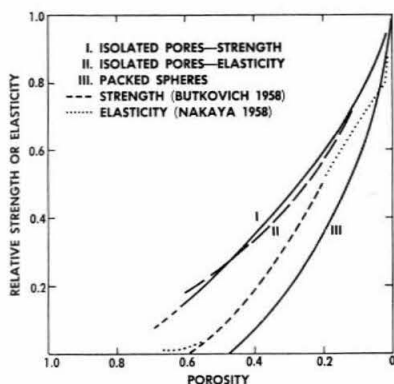


Fig. 4. Relative strength and elasticity, theoretical and experimental.

The validity of the assumption in this section can be demonstrated by computing the mechanical properties of snow from the density and comparing with experimental data. We have determined a theoretical relationship between density and the mechanical properties of firn and ice for two simple models: (1) a cubic packing of intergrowing spherical grains with continuous pore space, an approximation of firn, and (2) a continuous ice phase with isolated spherical holes, an approximation of glacier ice. Other models are, of course, possible. Details of the calculations will be presented in a forthcoming paper. Figure 4 shows the results. The two models give upper and lower bounds for the mechanical properties, and in particular both models predict accurately the rate of change of mechanical properties and structure of glacier material from in situ seismic measurements.

Compressive strength, work of disaggregation, and dielectric constant of snow at constant temperature are plotted against time (in hours) on bilog scales in Fig. 5. Even though there is a scatter in the slopes, the curves are nearly linear, a fact which suggests that the expected power relation is valid. In all cases the slopes indicate that $r \leq t^{1/5}$. We interpret the latter result to indicate that evaporation condensation and viscous flow processes are not the major causes of densification because they would lead to a much faster rate of growth of contact area. The evidence suggests that diffusion processes may control the growing together of ice grains and that these processes are responsible for so-called "regelation."

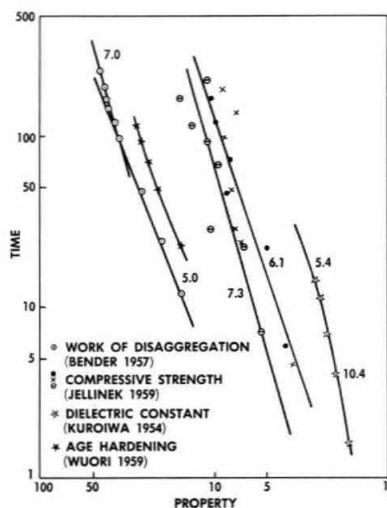


Fig. 5. The change of various physical properties of snow with time under constant pressure and temperature.

The actual flow mechanism in ice is more nearly plastic than viscous (Nye, 1952). If we use Orowan's (1949) assumption that ice behaves purely as a plastic substance with a definite yield stress, we may approximate the condition for flow in an assemblage of ice grains by the following equation:

$$k\phi = \sigma = \int \rho g dz \quad (10)$$

where

$$k = \text{yield stress}$$

and

$$\phi = \text{relative area of contact between grains } (\phi = 1 \text{ for pure ice})$$

If we assume the average density of near surface snow to be 0.40 g/cm^3 , a typical value for ϕ is 0.02 (from elasticity and strength data). Approximate values for k are 1 bar for shear stress and 2 bars for compressive stress (Orowan, 1949). Thus, flow at the contact points would begin at a depth of 50 or 100 centimeters, depending on whether the yielding resulted from shear or compressive stress, respectively. In either case it is clear that, to the extent that purely plastic behavior is a reasonable approximation to the flow of ice, we must expect flow to occur within the upper layers of snow and firn on the ice sheet. Pure ice will flow under the above assumptions, starting at approximately 20 meters. Since strain rate is proportional to at least the third power of stress (Glen, 1952), the importance of plastic flow increases with depth. When isolated bubbles are formed, the difference between load pressure and bubble pressure is the pertinent quantity for densification and this decreases with depth (Langway, 1958).

As grains grow together under constant load, the contact area increases, and the strain energy correspondingly decreases. However, in areas of accumulation the load on a given unit volume of snow is not constant but increases because of the accumulation of new snow above. As pressure on the grains increases, the motivations for material transport also increase—namely, the stress gradient for flow, chemical potential gradient for melting, and the stress or vacancy gradient (both included in the chemical potential gradient) for diffusion. If the accumulation is constant, the load on a given layer increases linearly with time. Therefore, if the stress on a given grain is to increase continually, the increase of contact area with time must be less than linear.

Data on strength as a function of density (11) are plotted as strength versus age in Fig. 3. The change of variables, from density to time, was done by stratigraphically dating the layers in the SIPRE deep pit. Also, the steady-state condition of constant accumulation makes it possible to compute the age directly from depth-density data (2). In situ the strength of snow, and therefore presumably the contact area, increases as $t^{0.84}$ until the critical density is reached; the resulting increase in pressure on the grains is approximately $t^{0.16}$. Beyond the critical density, strength increases as $t^{0.68}$, leading to a pressure increase of $t^{0.32}$. From the snow surface to the critical depth, stress on the grains is relieved primarily by grain rearrangement and packing. When the critical density is exceeded, stress can be relieved only by the grains growing together. The processes causing this growth proceed more slowly than grain packing, therefore the stress on the grains increases more rapidly after the critical density has been exceeded. It is apparent that both above and below the critical depth the actual stress on the ice grains increases much less rapidly than the total load. The continuous increase in intergranular contact area indicates that a truly "constant-stress" test is exceedingly difficult to perform. It is for this reason that conventional "creep" tests on snow, as reported in the literature, were not used in this study.

Compactibility

The deformation of snow caused by loading has been associated with a property called compressibility, compactibility, or viscosity, although usage of these terms in this regard has been inconsistent and ambiguous. Compressibility as defined in elastic and thermodynamic theory is not applicable to the nonelastic, irreversible process of snow densification. Because of the changing area of contact between grains and the continually increasing load on a given particle of snow the term viscosity in the context

of a physical property of the snow has no meaning. As used in the literature, it is more a property of the environment than the material. However, a parameter applying to the compaction of snow in situ is desirable, and we therefore define a steady-state compactibility in analogy to the definition of compressibility:

$$B = \frac{1}{v} \frac{dv}{d\sigma} \quad (11)$$

where B is the steady-state compactibility and the other variables are as defined before. We expect B to be a function of pore space and the mechanisms of densification.

Equation 11 looks innocuous enough, but it must be examined in detail in order not to be misunderstood and misapplied. It is to be considered a phenomenological relation between the variables as they are actually observed in a snow profile and not as a law of nature to be used in prediction, as for instance Hooke's law which it resembles. Both v and σ are time dependent; v does not respond instantaneously to a given σ , and σ is not applied instantaneously. To examine the nature of B , we write

$$dv(\sigma, t) = \frac{\partial v}{\partial \sigma} d\sigma + \frac{\partial v}{\partial t} dt$$

i. e., the snow reaches its present state by a combination of time-dependent and load-dependent processes. Since the only external force acting on the snow is owing to accumulation,

$$dv = \left(\frac{\partial v}{\partial \sigma} \frac{d\sigma}{dt} + \frac{\partial v}{\partial t} \right) dt$$

Since load can be determined from depth-density data alone while time cannot, we write the more useful form

$$\frac{dv}{d\sigma} = \left(\beta_1 + \frac{\beta_2}{A} \right)$$

where

$$\beta_1 = \frac{\partial v}{\partial \sigma}$$

$$\beta_2 = \frac{\partial v}{\partial t}$$

Previous authors would refer to β_1 as a compressibility, which it is not, and β_2/A as a viscosity, which it is not. However, β_1 is a stress modulus, and β_2 is a time modulus. The complexity of B is now apparent: It is a combined stress and time modulus. When accumulation is very small, time-dependent processes such as those previously considered dominate the densification process. When accumulation is large, the load-dependent processes dominate. In either case Eq. 11 is valid.

Under steady-state conditions the relative change in pore space is proportional to the change in stress caused by accumulation of new snow as stated in Eq. 1. The combination of Eqs. 1 and 11 provides a relation between compactibility and porosity n ; i. e.,

$$B = - \frac{m(v - v_i)}{v} = -mn \quad (12)$$

We may now regard the parameter m as a "compaction modulus." It is emphasized that the compaction modulus may be obtained from depth-density information alone since load is the integrated depth-density function. Thus, the compaction modulus is a useful parameter of snow and firn densifying in situ, particularly because the accumulation rate does not need to be known.

As a consequence of the steady-state conditions, we could equally well have expressed the results in terms of pressure or time. It is easy to show that

$$\begin{aligned} \frac{dv}{d\sigma} &= vB = -m(v - v_i) = -mv_i \epsilon = -mnv = -m(v_0 - v_i) e^{-m\sigma} \\ &= -m(v_0 - v_i) e^{-mAt} \end{aligned} \quad (13)$$

Thus the compactibility can be alternately defined as

$$\begin{aligned} B &= -mn = -m \frac{v_i}{v} \epsilon = -mv_i \rho \epsilon = -\frac{m}{v} (v_0 - v_i) e^{-mAt} \\ &= -\frac{m}{v} (v_0 - v_i) e^{-m\sigma} \end{aligned} \quad (14)$$

The load relationship is particularly interesting. As a consequence of Murnaghan's theory of finite strain (12) it may be shown that the pressure dependence of compressibility is

$$\beta = \beta_0 e^{-5\beta_0 \sigma} \quad (15)$$

when the compressibility β is defined as $(1/v_0)(dv/d\sigma)$ and β_0 is the compressibility at zero pressure. When compactibility is redefined analogously as $(1/v_0)(dv/d\sigma)$, call it B' , then

$$B' = -\frac{m}{v_0} (v_0 - v_i) e^{-m\sigma} = B_0 e^{-m\sigma} \quad (16)$$

which corresponds to the compressibility-pressure relation. It should be kept in mind that here load is a (linear) function of time, and is not hydrostatic. The parameters in Eqs. 15 and 16 are of the same orders of magnitude.

Significance of the Critical Density – Snow, Firn, Névé and Glacier Ice

Recognition of the critical density introduces the possibility of making a physical distinction between snow and firn, or névé. At present, the terms firn and névé are used interchangeably for material that is transitional between snow and glacier ice. These terms are not defined in terms of measured physical properties but are generally referred to as snow that is more than 1 year old. This definition implies (but does not specify) heavy summer melting to the extent that fresh winter snows may be clearly distinguished from the melt-altered snow cover of the previous year. In the nonsoaked facies of glaciers, especially in the dry-snow facies of Greenland or Antarctica, one cannot recognize such distinction even within a series of snow layers that may be 10 or more years old. However, use of the critical density as the point of subdivision between snow and firn, or névé, bases the distinction solely on the structure of the material. This distinction is independent of the time required for snow to exceed the critical density. However, it is also desirable to have a term for snow that has been wetted and yet has endured a complete summer melt season. Such material is clearly differentiated from new snow in the vicinity of the firn line regardless of whether or not it has exceeded the critical density. With these thoughts in mind, the following definitions are proposed:

Snow

Newly fallen snow consists of hexagonal crystals whose form depends on the conditions of their formation (13). The crystal shapes are rounded by the temperature-dependent diagenetic processes such as evaporation condensation, diffusion processes, and plastic flow. Within several days the snow consists of a loose aggregate of rounded grains whose physical properties change rapidly with time and increasing load. Most of the processes of material transport tend to spheroidize the grains and favor the growth of large grains at the expense of the small. Densification of snow proceeds primarily by readjustment and closer packing of grains simultaneously with the spheroidization process. The minimum porosity obtained by grain packing for dry snow is 36 to 40%, which is the same as the minimum values obtained by packing assemblages of spheres or sand grains. It lies approximately midway between loose (47.6%) and close (26.0%) packing values for spheres. When water is present, it not only fills pore space but permits tighter packing of grains. Snow conditions affect the critical density obtained in artificial compaction of snow (14). Snow is essentially an unconsolidated sediment.

Firn

Firn is snow that has been wetted by melt water, but that has survived at least one complete summer melt season and is distinguishable from the new snow overlying it. The term firn specifically carries the meaning "of last year," and the mechanisms of densification are not considered in its definition.

Névé

When the maximum densification of snow by packing is obtained at the critical density, there is a marked reduction in the rate of densification as revealed by the compaction modulus m . Densification beyond the critical density must proceed by mechanisms other than packing, such as plastic flow and molecular diffusion. These are time-dependent mechanisms. The term névé is applied to this material, and the adjective "wet" or "dry" should be used according to whether or not the material has been wetted during its history. Névé densifies by grains growing together to relieve stress. During this process pore spaces gradually become isolated and more spherical with steady reduction in the permeability to air flow. The entire process results in decreased surface area and consequently a decrease in free energy. The term névé is not defined in terms of the amount of time required to bring it into being but solely on the basis of the mechanisms by which it densifies. The range of névé extends from the critical density (maximum packing density) to the point where air permeability becomes zero. Névé is essentially "consolidated snow or firn."

Glacier Ice

When all the pores are isolated, air permeability becomes zero, and the névé becomes glacier ice. Densification continues primarily by plastic flow and molecular diffusion.

It is not a happy task to redefine terms that have been in use for a long time. However, the terms firn and névé have not been well defined; some meanings stress the time required to form the material, while others stress physical properties. The attempt here is to restrict one of them to the time aspect of the definition, while at the same time specifically stating that the material must have undergone melting. The other term is defined solely in terms of its structure. We thus have four terms to apply to "glacier-stuff": snow, firn, névé, and glacier ice. There has been no serious ambiguity in the use of the terms "snow" and "glacier ice." The definition of firn is now more specific than it has been. However, the definition of névé is new, and the terms "consolidated snow or firn" could be used in its place.

The concept of critical density, with its associated abrupt change in rate of densification, is not necessarily unique to snow and ice. Any sediment should be expected to densify initially by particle packing, with other processes taking the dominant role after packing is complete. This should be especially evident in uniformly deposited, deep ocean sediments that are free from disturbing influences of ground water and subareal processes. It is significant that the maximum recorded porosity for shale is generally in the range of 36 to 40%, and the minimum clay porosity is about 36% (15). Likewise the maximum porosity for sandstone is in the range of 36 to 51% with 37.7% being given most consistently as the upper limit; the minimum porosity for sand is about 30.2% (15). Thus it appears that the critical porosity is also the dividing line between clay and shale and between sand and sandstone. In summary, it seems that the critical porosity serves to distinguish between unconsolidated and consolidated sediments in general, as well as for the distinction between snow and névé.

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